Analysis of Fairness and Incentives of Profit Sharing Schemes in Group Buying

Feyza Merve Hafızoğlu and Sandip Sen

Department of Computer Science
University of Tulsa, Tulsa, OK 74104 USA
{feyza-hafizoglu,sandip}@utulsa.edu

Abstract. Payoff distribution within coalitions in group-buying environments, where a group of buyers pool their demands to benefit from volume discounts, is a well-studied problem. However, the general assumption in literature is unit demand, where every buyer needs one item. In the case of varying volume demands, both the valuation and the contribution of buyers will change. In this paper, we introduce the *variable demand group-buying game* with implied values, where the valuation of one item for the buyer is equal to the unit price, which the buyer can obtain by itself. Buyers with higher volumes of demand have lower valuation per unit. We consider scenarios where volume discounts kick in at multiple volume thresholds and investigate the effect of different profit sharing mechanisms in coalitions of buyers: proportional cost sharing based on volume demand and valuation, proportional profit sharing based on volume and contribution, and adjusted Clarke mechanism. All these mechanisms are efficient, budget-balanced, and individual rational. We evaluated these five payoff mechanisms on the following criteria: stability, incentive compatibility, and fairness. We introduce a fairness criteria that correlates with marginal contribution. Experimental results show that fairness and stability are difficult to satisfy simultaneously.

Key words: group-buying, variable demand, fairness, payoff distribution

1 Introduction

Electronic marketplaces offer great opportunities for sellers of products and services to reach out to and attract new prospective buyers. On the other hand, such markets also allow prospective buyers to search for opportunities to seek and find better products under possibly more advantageous contractual obligations. Interestingly, win-win situations can arise in such environments where sellers offer volume discounts to facilitate selling of larger quantities of products with resultant increase in total profits, and buyers forming "buying groups" by pooling their individual demands.

Forming "buying groups" can lower their per unit costs for both the sellers and buyers. From the perspective of the sellers, the order processing, shipping, manufacturing, and inventory costs decrease, the customers tend to buy more since some money will be left after volume discount, and the vendor can access to the revenue earlier than when it has sell to individual customers. From the perspective of the buyers, they save money by taking advantage of volume discounts.

Group-buying problem is well-studied in theory [1, 2, 3, 4, 5, 6] and practice [7, 8, 9]. The most popular cost distribution in real group-buying markets is equity: every buyer will pay the same amount per unit item. The cost distribution problem can also be determined by using auctions [1, 2, 3, 4]. The buyers can combine their demands in a bundle [3]. The common assumption in most studies that adopt a game-theoretical approach [1, 2, 4, 6] is unit demand. However, there are real-life group buying environments where the buyers need multiple items.

The purpose of this study is to understand the behavior of the profit sharing mechanisms for the buyers with varying demand sizes. When the number of items purchased by each buyer is different from each other, the contribution of each buyer will change dramatically with comparison to domains with unit demand. Hence a critical question arises: Who will pay how much? Should buyers make their payment based on the same price per unit? If not, why and what should price per unit for each buyer be? What are the desirable features of an ideal profit sharing mechanism? Do they differ from the desirable features of mechanisms designed for unit demand?

We study the resultant *variable demand group-buying game* scenarios, formally identifying novel aspects of the problem. In particular, the buyers are only required to express their volume demands and not their true valuation per item. From the volume demand expressed by a buyer, one can infer its minimal valuation per item, which is the price per unit item it had to pay if it was the lone buyer. Then, we identify desirable features of profit distribution schemes for buying groups: stability, incentive compatibility, and fairness. We find that the concept of fairness (the correlation between the marginal contribution of a buyer to the group and its profit) becomes more important, because of varying buyer demands. Although stability is a desirable criterion, a stable mechanism might not be attractive to buyers because it might be unfair in these kind of games of implicit valuation. For instance, if a buyer contributes more than the other buyers in a coalition, and gets zero profit, this buyer probably loses its faith in this mechanism. Another aspect of the comparison between stability and fairness is that in real life it is not easy to detect the stability of a mechanism (they need to try at all possible coalitions). On the other hand, a group of buyers who participate in a coalition can easily see the unfairness. In fact, humans strongly care about fairness in real life [10].

Section 2 introduces the variable demand group-buying game; Section 3 addresses the evaluation criteria of the mechanisms that are presented in Section 4. In Section 5, the mechanisms adapted to the variable demand group-buying game are discussed based on the criteria. Finally Section 7 concludes.

2 Group Buying Game

We introduce the *Variable Demand Group Buying Game* to represent and reason about scenarios where buyers who need multiple items of one type or different amounts of some particular service. For instance, in group travel domain, e.g. travel deals in GroupOn[1], a group of people who are interested in travelling to the same destinations can combine their travel demands and benefit from group discounts. The unit price is the price per person for the travel and determined by the travel agency: the unit price monotonically decreases as the total demand increases. The number of travelers for each buyer will vary: some buyers might be couples, while others travel with their families or friends.

The unit price for two buyers, one traveling with a spouse and another travelling with a group of friends, is same in practice. However, the contribution of the second buyer travelling with a group of friends, in reducing per person cost is more than the couple. If this crowded friend group would not attend the travel, the couple and the rest of the buyers might not benefit from such a high discount and end up paying more for their trip. Furthermore, the expected discount for the buyers might differ based on their demand size. Buyers with lower demands, e.g. a couple, is ready to pay for higher unit prices than the unit price for the buyer traveling with a group of friends, because the couple cannot get better deals by themselves. On the other hand, the buyer travelling with a friend group will expect a more discounted rate per group member.

Since our primary goal is to describe the features of ideal profit sharing mechanisms for group-buying markets with multiple demands, we make some simplifying assumptions about the market. We are interested in the buyers' benefits assuming that sellers decided their price schedules to maximize their own profit. We do not address the issue of competition between the sellers. The price schedule of the seller is assumed to be public information.

2.1 Variable Demand Group-buying Game

Variable demand group-buying game is a non-convex game (see Appendix for proof) consisting of buyers, price schedule, profit sharing scheme, and the utility function.

Variable Demand Group-buying Game: The group-buying game is a tuple $\langle G = (N, \mathbf{n}, p) \rangle$

- $N = \{b_i\},\ i = [1 \ldots l]$ is a set of l buyers;
- $\mathbf{n}$ is the vector of agent demands, where n_i is the demand volume of the i^{th} buyer.
- $p : \mathbb{N} \mapsto \mathbb{R}$ is the price schedule of the seller;

In addition, we consider different *payment schemes* which determine the payment to be made by each buyer given a game G: $S : \mathbf{n} \times p \mapsto \mathbb{R}$. A given payment

[1] http://www.groupon.com/

scheme, in turn, decides the profit of each of the buyers from this particular purchase.

Sellers and price schedules : We consider one type of item and the profit distribution of one seller. The profit sharing scheme is considered to be generic to be used for any price schedule of volume discounts. There are two constraints for a price schedule, $p(n)$, given n, the total number of items required by the buyers (also referred to as the total demand): when $n_1 \geq n_2$ we require (1) $p(n_1) \leq p(n_2)$, i.e., the price schedule is monotonically non-increasing, and (2) $n_1\ p(n_1) \geq n_2\ p(n_2)$, i.e., increasing demand will not lower the total cost or business volume.

Buyers : The population of buyers is denoted as $N = \{b_1, \ldots, b_l\}$. The buyer i, denoted as b_i, only reports its volume demand, n_i, where $n_i \geq 1$. Although the true valuation of b_i, i.e., $p(n_i^*)$, is unknown, the minimum valuation of a unit item for the buyer is implied as the unit price that it can obtain by itself, i.e., $p(n_i)$. Hence, the implied valuation of b_i's demand is $n_i p(n_i)$.

Coalition : Let's say N is a set buyers, $\{b_1, \ldots, b_l\}$, in a coalition, denoted as C_N. The profit of the coalition becomes the total valuation minus the total cost of the coalition

$$u(C_N) = \sum_{i \in C_N} n_i p(n_i) - n_T p(n_T) \qquad (1)$$

where n_T is the total volume, i.e., $\sum_{i \in C_N} n_i$. b_i's profit becomes its valuation minus what it pays, i.e., $n_i\ p(n_i) - \text{pay}_i$.

Coalition Formation : In this game, the grand coalition is the optimal coalition because the buyers are ready to make a payment based on their valuation. Considering the first constraint of the price schedule, the price per unit item that existing members of the coalition will pay cannot increase by including other buyers. That means the profit of the coalition never decreases by adding another buyer. Furthermore, the utility of the coalition increases as we keep adding buyers into it, i.e., $u(C_S) \leq u(C_N)$ given that $S \subseteq N$. The contribution of b_i is defined as the difference in the utilities of the coalitions with, C_N, and without, $C_{N/\{i\}}$, b_i:

$$con_i(C_N) = u(C_N) - u(C_{N/\{i\}}) = n_i p(n_i) + (n_T - n_i) p(n_T - n_i) - n_T p(n_T). \qquad (2)$$

Figure 1 shows a coalition with three buyers: $\{b_1, b_2, b_3\}$ for the given price schedule. Each bar represents one buyer, where the width of the bar shows the expressed demand volume, n_i, and the height is the buyer's valuation per unit, $p(n_i)$. Thus, the area of the bar is the total valuation of b_i for its demand, i.e., $n_i p(n_i)$. Buyers are sorted in descending order based on their value per unit to better visualize the big picture (this is not key for the working of the system as all buyers are finally included in the grand coalition, the order of inclusion does not matter).

Total value of the coalition, i.e., $\sum_{i \in N} n_i\ p(n_i)$, is the area consisting of three bars. According to the price schedule, the unit price is 6 (specified by the dashed line) for demand volume 6, and the cost of the coalition is the area under the

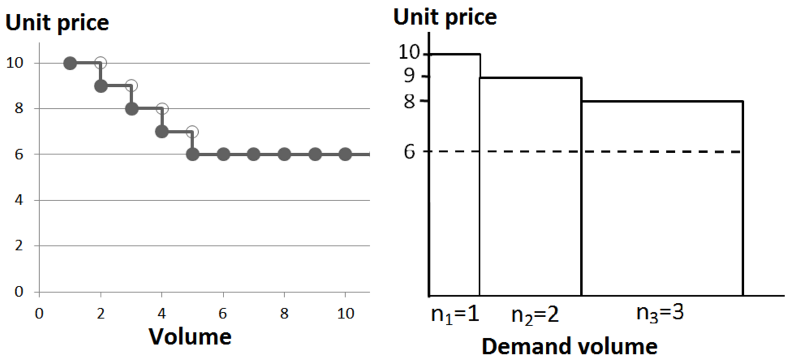


Fig. 1. Price schedule and a coalition with three buyers

dashed line. Thus the area above the dashed line is the profit of the coalition. Our goal is to understand the desirability of different mechanisms for sharing this profit between buyers based on their expressed demands. Which buyer desires more profit? For Figure 1, should it be b_3, because of its contribution in reducing the per unit cost, or should it be b_1, with its higher value per unit item?

3 Evaluation Criteria

We use the following criteria for evaluating profit sharing mechanisms:

Individual Rationality: The net utility of participants should be nonnegative, i.e., $\forall i \in C_N$, $u_i(C_N) \geq 0$.

Budget-balanced: The coalition should get exactly what it pays for, i.e., $\sum_{i \in C_N} \text{pay}_i = n_T p(n_T)$, where pay_i is the payment of b_i.

Incentive Compatibility: Participants cannot gain more utility by misrepresenting their valuations or, as in this case, their demands. The valuation of b_i is implicitly determined based on n_i and the price schedule. A profit sharing mechanism will be incentive compatible if b_i cannot increase its utility by speculating on n_i, i.e., $n_i = n_i^*$, where n_i^* is the true demand of agent b_i.

Efficiency: The social welfare, the sum of the utility of the participants, should be maximized, i.e., $\sum_{i \in C_N} u_i(C_N) + u_s$ where u_s is the utility of the seller.

Stability/Core: None of the subgroups of the coalition can gain more utility by leaving the current coalition and forming a new coalition, i.e., $\forall S \subseteq N, \sum_{i \in C_S} u_i(C_N) \geq u(C_S)$.

Fairness: In this game, fairness is measured in terms of the correlation between the contribution of a buyer and its profit. The idea is that b_i should receive a share of the profit proportional to its (non-negative) contribution to the coalition. The rationale for this fairness measure is that if b_i is not compensated for its contribution then it is being "underpaid" or not recognized. Also, such an "unfair" compensation may incentivize b_i to leave the coalition, in which case the value or profit of the entire coalition may decrease.

Furthermore, people may be able to recognize unfair treatment and avoid such environments in real life. In our domain of group-buying, given the public price schedule in a market, a buyer can approximately compute its contribution by estimating total demand (the individual demands of other buyers are not needed). In case of an unfair profit distribution, this buyer will lose its trust in the process and refuse further participation. We use two different fairness metrics:

1. *Pearson correlation coefficient* between the contribution and the profit of a buyer in a coalition
2. The deviation in the profit distributions of mechanisms from the fairest mechanism, *profit sharing proportional to contribution.*

4 Mechanisms

We introduce five mechanisms with different characteristics to understand their behavior in this setting. Even though they distribute profit (or cost) based on simple heuristics, their approaches are diverse enough to highlight the desirable features of mechanisms in the presence of varying demands.

1. **Cost Sharing Proportional to Volume:** The total cost is distributed proportional to demand volumes, i.e., $\text{pay}_i = n_i p(n_T)$. b_i's utility (share of profit) is
$$u_i(C_N) = n_i p(n_i) - n_i p(n_T) = n_i(p(n_i) - p(n_T)) \tag{3}$$
by paying $n_i p(n_T)$. Everybody obtains the same per unit price regardless of their demand volume. It is extremely advantageous for the buyers with lower demands because of their higher valuations. The profit of buyers only depends on their valuation and the per unit price for the coalition. This mechanism is quite popular in real group-buying markets.
2. **Cost Sharing Proportional to Valuation:** The cost is distributed proportional to the valuation of b_i. This mechanism is not explicitly favoring any buyer type. b_i's utility is
$$u_i(C_N) = n_i p(n_i) - \frac{n_i p(n_i)}{\sum_{i \in C_N} n_i p(n_i)} n_T p(n_T). \tag{4}$$
3. **Profit Sharing Proportional to Volume:** The profit of the coalition, i.e. $u(C_N) = \sum_{i \in C_N} n_i p(n_i) - n_T p(n_T)$, is distributed proportional to volume. b_i's utility is
$$u_i(C_N) = u(C_N) \frac{n_i}{\sum_{i \in C_N} n_i}. \tag{5}$$
This mechanism is more advantageous to buyers with larger demands. Although it guarantees that the buyers with smaller demands will obtain a non-negative profit, their share of profits will be smaller.

4. **Profit Sharing Proportional to Contribution:** Contributions of buyers fairly determine their profits:

$$u_i(C_N) = \frac{\text{con}_i(C_N)}{\sum_{i \in C_N} \text{con}_i(C_N)} u(C_N). \tag{6}$$

5. **Adjusted Clarke Mechanism[2]:** This mechanism, similar to the profit sharing proportional to contribution mechanism, shares the utility based on the marginal contribution of buyers, but the actual payment is derived through a two-stage calculation. First a tentative payment of a coalition member is determined by reducing its valuation by its contribution. At this point the total sum of the buyer payments falls below the total cost of the coalition, $n_T p(n_T)$. Then the shortfall, i.e., the rest of the cost, is equally divided between the buyers to achieve budget balance. Hence,

$$\text{pay}_i = \begin{cases} n_i\, p(n_i) & \tau_i + \mu > n_i\, p(n_i) \\ \tau_i + \mu & \text{otherwise} \end{cases} \tag{7}$$

where $\tau_i = n_i p(n_i) - \text{con}_i(C_N)$ and μ is a positive constant number which makes the solution budget balance.

Using Shapley value [11] is an ideal approach to assess the marginal contribution, we did not examine this mechanism because of its exponential computational complexity. In addition to that using Shapley value for profit sharing is advantageous in the case of convex games because the solution is guaranteed to be in the core. Since variable demand group-Buying game is a non-convex game, we cannot take advantage of this mechanism in terms of stability.

5 Analysis

Budget-balanced: Three of five mechanisms, cost sharing proportional to volume and valuation, and adjusted Clarke mechanism, distribute the total cost of the coalition among the buyers. The others, namely profit sharing proportional to volume and contribution, compute the profit by subtracting the total cost from the total valuation to make sure that total cost is allocated to pay the seller. Hence, the coalition pays exactly what it gets from the buyers. The other relevant fact is that in the game there is no external agency that transfers utility to buyers or sellers. Therefore, all mechanisms are budget-balanced by definition.

Individual Rationality: Profit sharing proportional to volume and contribution mechanisms distribute the non-negative profit among buyers and are individual rational by definition. Cost sharing proportional to volume and valuation mechanisms compute what percentage of the total valuation of each buyer will be paid. In other words, the per unit price assigned to buyers is always lower than their valuation of the unit item. That is why these four methods are individually rational. Finally, adjusted Clarke mechanism is explicitly designed

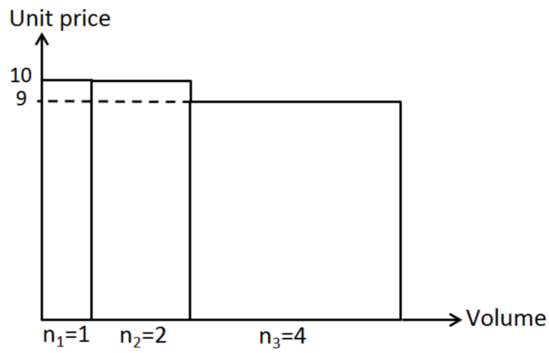


Fig. 2. A critical scenario for fairness

to be individually rational as $\text{pay}_i \leq n_i p(n_i)$, i.e., payment is less than or equal to valuation.

Efficiency: Since the grand coalition is optimal yielding a maximum profit in the game to the entire set of buyers and the seller, as all buyers are included and profit is monotonically non-decreasing in the group of buyers; all mechanisms are efficient because of this nature of the game.

Stability: The core of the game is not empty and out of five mechanisms only the cost sharing proportional to volume mechanism is in the core (see Appendix for proof). During the test runs with other mechanisms and while varying price schedules and demand volumes, subcoalitions rarely stand to gain by leaving the grand coalition, and even then the gain is very small and may not warrant deviation in practice because of knowledge requirements and other contributing factors like the cost of forming subcoalitions, etc. The rareness of instability is largely due to the fact that grand coalitions typically get the minimum price per unit and hence by far the higher total profit or social welfare.

Incentive Compatibility: None of five mechanisms is incentive compatible.

5.1 Fairness Analysis

Before discussing fairness, we show an example scenario where the cost sharing proportional to volume mechanism, which satisfies the stability, produces an unfair profit distribution. Figure 2 shows a coalition of buyers $\{b_1, b_2, b_3\}$ given the price schedule: $p(n) = 10$ when $n < 4$, and $p(n) = 9$ for $n \geq 4$. The demand volumes are 1, 2, and 4, respectively. The implied per unit valuations of buyers, i.e. $p(n_i)$, are 10, 10, and 9, respectively. The discounted per unit price for the coalition is 9 (dashed line). In this case, the total valuation of the coalition is 66, the total cost of the coalition (the total area under the dashed line) is 63, and the total profit is 3 (the area above the dashed line). Finally, the contributions of buyers are 1, 2, and 3, respectively.

Cost sharing proportional to volume mechanism distributes the profits as 1, 2, and 0, respectively. Although b_1 and b_2 cannot reduce the price without b_3, b_3 has no share from the profit in return for having larger contribution. Furthermore, b_3 does not need to participate in a coalition unless the per unit price drops below its implied value. Hence, b_3 has no motivation to stay in the coalition and can be unhappy if it can infer or estimate this unfair profit distribution. The natural expectation of such buyers would be to gain nonzero profit in return for their contribution to the profits of others.

Table 1. Profit distributions for the coalition in Figure 2

Mechanism	$u(b_1)$	$u(b_2)$	$u(b_3)$
Cost Sharing Prop. to Volume	1	2	0
Cost Sharing Prop. to Valuation	0.45	0.91	1.64
Profit Sharing Prop. to Volume	0.43	0.86	1.71
Profit Sharing Prop. to Contribution	0.5	1	1.5
Adjusted Clarke	0	1	2

Table 1 presents the profit distributions for this scenario. It can give an idea about how mechanisms treat buyers with different demands. Cost sharing proportional to volume mechanism produces a less than fair share of profits for buyers with larger demands and having a significant contribution. The positions of three mechanisms, namely, cost sharing proportional to valuation, profit sharing proportional to volume, and contribution, are somewhat similar: every buyer gets a share from the profit somewhat proportional to their contributions. Interestingly, the adjusted Clarke tax mechanism too yields an unfair profit distribution, though the under-rewarding is less conspicuous. When using the adjusted Clarke tax, b_1 suffers from the unfairness. Interestingly, the two sufferers for the two mechanisms, b_1 and b_3, are buyers at two extreme demands: one with a relatively large demand volume and the other with a relatively small demand.

From the representative example, we conclude that in such critical coalitions, adjusted Clarke mechanism is advantageous for buyers like b_3 while it severely deprives buyers like b_1 of the profit. Precisely the opposite happens with cost sharing proportional to volume mechanism where buyers with high marginal contributions are not rewarded enough.

Experimental evaluation of fairness: To gauge the fairness of different mechanisms over a large set of scenarios, we implemented a simple group-buying market. The number of buyers is fixed in a run to understand the behavior of the mechanisms. However, the demand volumes are uniformly distributed within the range between 1 and 10, i.e., $n_i \in U[1, 10]$. 100 price schedules are randomly generated: each price schedule is tested with 100 randomly generated volume demands for a group of buyers. Therefore 100× 100=10000 different coalition settings (a unique combination of demand volumes of buyers and the price schedule) are tested with a fixed number of buyers to report the averages.

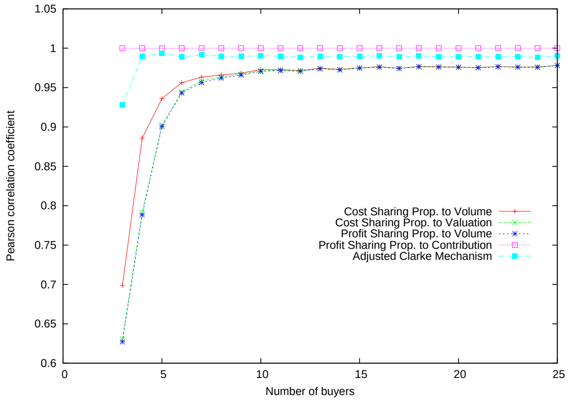


Fig. 3. Correlation between contribution and profit

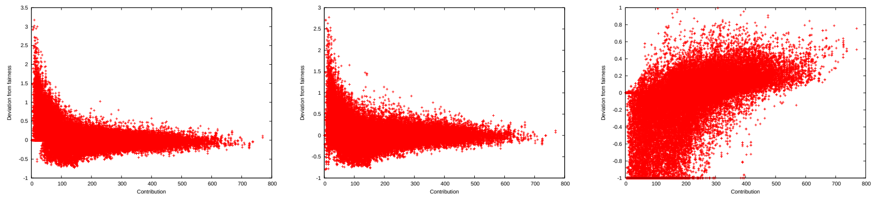
Correlation coefficient: Initially, Pearson correlation coefficients of each mechanism's profit distribution is computed for each coalition and then the average coefficients are calculated within each mechanism. This process is repeated for different numbers of buyers.

Figure 3 shows the correlation coefficients of mechanisms for varying coalition sizes. The closer the correlation coefficient is to 1, the fairer is the mechanism. We observe that the correlation coefficients of all mechanisms gradually increases up to coalition size of 15 and then stabilize with small oscillations. This result illustrates how the impact of one buyer is more remarkable in a small coalition. The unfairness of mechanisms is ameliorated in larger groups.

Apart from the profit sharing proportional to contribution (fairest by definition) the next fairest mechanism is the adjusted Clarke mechanism. For lower coalition sizes (up to 10), the cost sharing proportional to volume mechanism surprisingly produces fairer profit sharing than those produced by the cost sharing proportional to valuation and profit sharing proportional to volume mechanisms. For coalition sizes larger than 10, these three mechanisms have the same fairness attitude. One final point is that the correlation coefficients of these mechanisms do not converge to the same value, i.e. the relative order of fairness is preserved in larger groups even if the lack of fairness is mostly eliminated.

Deviation from fairness: The second metric measures how much mechanisms deviate from a fair distribution. Profit sharing proportional to contribution mechanism is again, therefore, the standard against which other mechanisms are evaluated. Let the fair profit for b_i be u_i^f and the profit distributed by mechanism m to b_i be u_i^m, then the relative deviation of mechanism m for b_i's profit is calculated $\frac{u_i^f - u_i^m}{u_i^f}$.

Figure 4 depicts the deviation of mechanisms from fairness for varying contribution levels in coalitions of seven buyers. We do not include the plot for the profit sharing proportional to volume mechanism because its behavior is very similar to cost sharing proportional to valuation mechanism. Figure 4 con-



(a) Cost Sharing Prop. to Volume (b) Cost Sharing Prop. to Valuation (c) Adjusted Clarke

Fig. 4. Contribution vs. deviation from fairness in coalitions consist of seven buyers

firms the observations from using the Pearson correlation coefficient but more clearly depicts in which cases they deviate from being fair. The general trend is that the magnitude of the deviation reduces as the contribution increases. Cost sharing proportional to volume (Figure 4(a)) and valuation (Figure 4(b)) mechanisms frequently reward lower contributors at a disproportionately high rate and deviate both negative and positive for different contribution levels. However, adjusted Clarke mechanism (Figure 4(c)) tends to provide gradually higher rewards to higher contributors. In addition to this, the absolute sum of magnitude of the deviation is clearly higher in adjusted Clarke's case.

The fairness metrics indicate that cost sharing proportional to volume and adjusted Clarke mechanisms diverge most from the fair distribution. Comparatively, the other mechanisms are more balanced for varying coalition sizes and contribution levels. One can argue that the fairness metrics we proposed are not powerful enough to demonstrate that cost sharing proportional to volume and adjusted Clarke mechanisms favoring two opposite buyer types. To demonstrate this claim, we compare the profits for varying volume demands.

Figure 5 shows the average profit distribution for varying demand volumes in coalitions of 7 buyers. Adjusted Clarke mechanism is advantageous for buyers with larger demands and less profitable for buyers with smaller demands. Contrarily, cost sharing proportional to volume mechanism produces relatively higher profits for buyers with smaller demands and lower profits for buyers with larger demands. The other mechanisms produce profit distributions between these two. However, the order of being advantageous for buyers with larger demands between mechanisms is preserved for increasing demand volumes.

6 Prior Work

Our study differs from prior work based on two dimensions: unit demand and auctions. Unit demand is the common assumption in group-buy auction games [1, 2, 4, 6], where buyers report their bids in auctions. We believe that implied value better corresponds to the true valuation of the buyer with comparison to bids in group-buy auctions because a buyer can only slightly increase

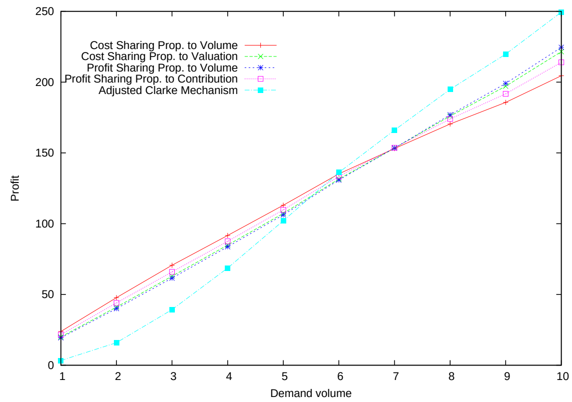


Fig. 5. Profit distributions in coalitions of seven buyers

its utility by deviating from its true valuation by reporting higher demand volumes (it will pay more for more items which cause a decrease in utility) or lower demand volumes (smaller demand volumes cause a decrease in the total value obtained from items) based on simulations. It is also practical way to infer valuations in case of lack of an auction mechanism. To the best of our knowledge, variable demand group-buying game is the first study that is specifically designed for the settings where buyers have varying demand volumes and does not require a bid. Furthermore, this study addresses the significance of fairness in particular for real-life group-buying environments. Since, humans are not purely self-interested [10] and fairness is an important criterion for them.

Group-buying markets are studied from social and economic perspectives. Lu and Boutilier [12] studied the problem of matching of buyers and sellers where a group of buyers with varying preferences over sellers to benefit volume discounts and investigate the computation of buyer-welfare maximizing matchings. Walter [5] studied trust in group-buy environment, where trust is used to model the similarity in preferences. Erdoğmuş and Çiçek [8] empirically investigate the behavior of buyers in group-buying markets by conducting interviews. Their results show that buyers are mostly attracted by the price advantage and discount amount and have complaints about the discriminatory and dishonest behavior of the service providers, which can be considered unfair. Liao *et al.* [7] analyze the customer behavior in online group-markets by using clustering analysis and rule generation. Kaufmann and Wang [9] analyze the number of orders in group-buy auctions over time for `Mobshop.com` products list. They characterize three different effects in the auction: positive participation externality effect (the number existing orders has a positive effect on prospective orders placed), price-drop effect (orders increase more when current price drops), auction-ending effect (more orders have been placed during the final period). The results of these social studies can be helpful for sellers to provide better service to customers and increase their profit.

7 Conclusion

We seek an answer for the question "How the profit should be distributed?" in the presence of buyers with varying demand sizes. As seen in the group-buy travel example, there is a critical issue that should be investigated to empower both sellers and buyers and increase social welfare. We introduce the variable demand group-buying game and define the implied value per item as the per unit price which can be obtained by the buyer itself. We identify several desirable features of payoff distribution schemes in the group-buy situation. In particular, we highlight the importance of the fairness issue by analyzing representative critical scenarios.

We propose and evaluate five profit distribution mechanisms in terms of the identified criteria. We observe that even though cost sharing proportional to volume is the only mechanism that is stable, it poorly distributes profit in terms of fairness by under-rewarding buyers having higher implied values. This indicates that satisfying stability and incentive compatibility criteria does not guarantee desirable coalitions in the eyes of buyers. On the other extreme, adjusted Clarke mechanism charges relatively higher payments to the buyers with lower contributions to subsidize the budget deficiency that arises from decreasing the payments of buyers with higher contributions.

Our findings suggest that the profit sharing should be carefully managed in the group buying markets with varying demand volumes. To avoid unfair situations, which will cause losing customers, an ideal profit sharing mechanism should not have a position which is explicitly advantageous for certain type of buyers, e.g. having higher implied value for per unit in cost sharing proportional to volume mechanism, and having high contributions in adjusted Clarke mechanism. Rather, a mild manner should be adopted to achieve fairer profit distributions based on the diversity of the buyers' attributes, e.g. implied value per unit, contribution, and demand volume.

References

1. Chen, J., Chen, X., Kauffman, R., Song, X.: Cooperation in group-buying auctions. In: System Sciences, 2006. HICSS '06. Proceedings of the 39th Annual Hawaii International Conference on. Volume 6. (January 2006) 121c
2. Li, C., Rajan, U., Chawla, S., Sycara, K.: Mechanisms for coalition formation and cost sharing in an electronic marketplace. In: Proceedings of the 5th international conference on Electronic commerce. ICEC '03, New York, NY, USA, ACM (2003) 68–77
3. Li, C., Sycara, K., Scheller-Wolf, A.: Combinatorial coalition formation for multi-item group-buying with heterogeneous customers. Decision Support Systems **49**(1) (2010) 1 – 13
4. Matsuo, T., Ito, T., Shintani, T.: A volume discount-based allocation mechanism in group buying. In: Data Engineering Issues in E-Commerce, 2005. Proceedings. International Workshop on. (april 2005) 59 – 67

5. Walter, F.E.: Trust as the basis of coalition formation in electronic marketplaces. Advances in Complex Systems (ACS) **14**(02) (2011) 111–131
6. Yamamoto, J., Sycara, K.: A stable and efficient buyer coalition formation scheme for e-marketplaces. In: Proceedings of the fifth international conference on Autonomous agents. AGENTS '01, New York, NY, USA, ACM (2001) 576–583
7. Liao, S., Chu, P., Chen, Y., Chang, C.C.: Mining customer knowledge for exploring online group buying behavior. Expert Systems with Applications **39**(3) (2012) 3708 – 3716
8. İrem Eren Erdoğmuş, Çiçek, M.: Online group buying: What is there for the consumers? Procedia - Social and Behavioral Sciences **24**(0) (2011) 308 – 316
9. Kauffman, R.J., Lai, H., Ho, C.T.: Incentive mechanisms, fairness and participation in online group-buying auctions. Electronic Commerce Research and Applications **9**(3) (2010) 249 – 262
10. Jong, S.d., Tuyls, K., Verbeeck, K.: Artificial agents learning human fairness. In: Proceedings of the 7th international joint conference on Autonomous agents and multiagent systems. (2008) 863–870
11. Shapley, L.S.: A value for n-person games. Contributions to the Theory of Games **2** (1953) 307–317
12. Lu, T., Boutilier, C.E.: Matching models for preference-sensitive group purchasing. In: Proceedings of the 13th ACM Conference on Electronic Commerce. EC '12, New York, NY, USA, ACM (2012) 723–740

Appendix:

Proof 1. Variable demand group-buying game is a non-convex game: A game is convex when $\forall_{S,T}\ u(C_{S\cup T}) \geq u(C_S) + u(C_T) - u(C_{S\cap T})$ is satisfied. We will prove that variable demand group-buying game is not convex by giving a counter example. Let's say $S = \{b_1, b_2, b_3, b_4\}$ and $T = \{b_1, b_2, b_3, b_5, b_6\}$ are two sets of buyers and every b_i has a demand of 1. $S \cap T = \{b_1, b_2, b_3\}$ and $S \cup T = \{b_1, b_2, b_3, b_4, b_5, b_6\}$.

$$p(n) = \begin{cases} 10 & n < 4 \\ 9 & n = 4 \\ 8 & n > 4 \end{cases} \tag{8}$$

Given the price schedule $p(n)$, the utilities become: $u(C_S) = 4$, $u(C_T) = 10$, $u(C_{S\cup T}) = 12$, and $u(C_{S\cap T}) = 0$. When we set these values into the convex game criteria $12 \geq 4 + 10 - 0$, the condition is not satisfied. Hence, variable demand group-buying game is a non-convex game.

Proof 2. Cost sharing proportional to volume mechanism is in the core:

$$\begin{aligned} \forall S \subseteq N, \sum_{i\in C_S} u_i(C_N) &\geq u(C_S) \\ \sum_{i\in C_S} \big[p(n_i) - p(n_T)\big] &\geq \sum_{i\in C_S} \big[p(n_i) - p(n_S)\big] \\ p(n_S) &\geq p(n_T) \end{aligned} \tag{9}$$